

# Observation of $\bar{B} \rightarrow D^{(*)}\pi^+\pi^-\ell^-\bar{\nu}$ decays in $e^+e^-$ collisions at the $\Upsilon(4S)$ resonance

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We report on measurements of the decays of  $\bar{B}$  mesons into the semileptonic final states  $\bar{B} \rightarrow D^{(*)}\pi^+\pi^-\ell^-\bar{\nu}$ , where  $D^{(*)}$  represents a  $D$  or  $D^*$  meson and  $\ell^-$  is an electron or a muon. These measurements are based on  $471 \times 10^6$   $B\bar{B}$  pairs recorded with the BABAR detector at the SLAC asymmetric  $B$  Factory PEP-II. We determine the branching fraction ratios  $R_{\pi^+\pi^-}^{(*)} = \mathcal{B}(\bar{B} \rightarrow D^{(*)}\pi^+\pi^-\ell^-\bar{\nu})/\mathcal{B}(\bar{B} \rightarrow D^{(*)}\ell^-\bar{\nu})$  using events in which the second  $B$  meson is fully reconstructed. We find  $R_{\pi^+\pi^-} = 0.067 \pm 0.010 \pm 0.008$  and  $R_{\pi^+\pi^-}^* = 0.019 \pm 0.005 \pm 0.004$ , where the first uncertainty is statistical and the second is systematic. Based on these results, we estimate that  $\bar{B} \rightarrow D^{(*)}\pi\pi\ell^-\bar{\nu}$  decays, where  $\pi$  denotes both a  $\pi^\pm$  and  $\pi^0$  meson, account for up to half the difference between the measured inclusive semileptonic branching fraction to charm hadrons and the corresponding sum of previously measured exclusive branching fractions.

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The semileptonic decays of  $B$  mesons to final states containing a charm quark allow a measurement of the magnitude of the CKM matrix element  $[1, 2] |V_{cb}|$ , a fundamental parameter in the standard model (SM) of particle physics that plays an important role in unitarity tests sensitive to physics beyond the SM [3]. Determinations of  $|V_{cb}|$

from inclusive semileptonic decays  $\bar{B} \rightarrow (X_c)\ell^-\bar{\nu}$ ,<sup>1</sup> where the hadronic state  $X_c$  is not reconstructed, and those from the exclusive semileptonic decays  $\bar{B} \rightarrow D^*\ell^-\bar{\nu}$  and  $\bar{B} \rightarrow D\ell^-\bar{\nu}$ , differ by nearly three

<sup>1</sup> Throughout this Letter, whenever a decay mode is given, the charge conjugate is also implied.

standard deviations ( $3\sigma$ ), as discussed on page 1208 of Ref. [4]. The measured exclusive  $\bar{B} \rightarrow X_c \ell^- \bar{\nu}$  decays,  $\bar{B} \rightarrow D^{(*)} \ell^- \bar{\nu}$ ,<sup>2</sup>  $\bar{B} \rightarrow D^{(*)} \pi \ell^- \bar{\nu}$ , and  $\bar{B} \rightarrow D_s^{(*)+} K^- \ell^- \bar{\nu}$  [4], account for only  $85 \pm 2\%$  [5] of the inclusive rate for semileptonic  $\bar{B}$  decays to charm final states. The decay modes measured in this Letter account for part of this difference. They also provide experimental information needed to quantify background-related systematic uncertainties in measurements of  $\bar{B} \rightarrow D^{(*)} \tau \bar{\nu}$  decays, which are sensitive to new physics contributions. A measurement [6] of these decays shows a  $3.4\sigma$  deviation from the SM, and independent measurements [7, 8] also exceed SM expectations.

We search for semileptonic decays of a  $B$  meson to a  $D$  or  $D^*$  meson and two additional charged pions, and measure branching fraction ratios  $R_{\pi^+\pi^-}^{(*)} = \mathcal{B}(\bar{B} \rightarrow D^{(*)} \pi^+ \pi^- \ell^- \bar{\nu}) / \mathcal{B}(\bar{B} \rightarrow D^{(*)} \ell^- \bar{\nu})$  relative to the topologically similar decays  $\bar{B} \rightarrow D^{(*)} \ell^- \bar{\nu}$ . The results are based on the complete sample of  $e^+e^-$  collision data collected at the  $\Upsilon(4S)$  resonance with the *BABAR* detector at the SLAC PEP-II storage ring, corresponding to  $471 \times 10^6$   $B\bar{B}$  decays ( $454 \text{ fb}^{-1}$  [9]). An additional  $40 \text{ fb}^{-1}$  sample, collected at center-of-mass (CM) energies just below the  $B\bar{B}$  threshold, is used to verify the modeling of background from  $e^+e^- \rightarrow f\bar{f}(\gamma)$  continuum processes with  $f = u, d, s, c, \tau$ .

The *BABAR* detector, as well as the reconstruction and particle identification algorithms, are described in detail elsewhere [10]. The analysis uses Monte Carlo (MC) simulated event samples to determine efficiencies and to model backgrounds. Simulated  $B\bar{B}$  decays are produced with the EvtGen [11] generator, with final-state radiation described using the PHOTOS [12] program. Continuum  $e^+e^- \rightarrow q\bar{q}$  events are generated with the JETSET [13] program, and  $e^+e^- \rightarrow \tau^+\tau^-$  events with the KK2F [14] program. The world averages quoted in Ref. [4] are used for branching fractions and form factor parameters. The GEANT4 [15] package is used to model the detector and detector response.

The intermediate process through which  $D^{(*)} \pi^+ \pi^-$  states arise in semileptonic  $B$  decays is unknown. We consider production via (1) 3-body phase-space decays,  $X_c \rightarrow D^{(*)} \pi \pi$ , (2)  $X_c \rightarrow D^{(*)} f_0(500)$  decays with  $f_0(500) \rightarrow \pi \pi$ , (3) sequential decays  $X_c \rightarrow Y_c \pi$  followed by

$Y_c \rightarrow D^{(*)} \pi$ , and (4)  $X_c \rightarrow D^{(*)} \rho$  decays with  $\rho \rightarrow \pi \pi$ , where  $X_c$  is one of  $D_1(2420)$ ,  $D(2S)$ , or  $D^*(2S)$ , and  $Y_c$  is one of  $D_1(2430)$ ,  $D_0^*$ , or  $D_2^*$ . The  $D^{(*)}(2S)$  states are the first radial excitations of the ground state  $D^{(*)}$  mesons, and are modeled as in Ref. [5]. Our nominal signal model consists of 3-body phase-space  $X_c \rightarrow D^{(*)} \pi \pi$  decays with an equal mix of  $X_c$  mesons.

We reconstruct events of the type  $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B\bar{B}$ . One of the  $B$  mesons ( $B_{\text{tag}}$ ) is fully reconstructed in a hadronic final state. To reconstruct a  $B_{\text{tag}}$  candidate, a seed (one of  $D^{(*)}$ ,  $D_s^{(*)+}$ , or  $J/\psi$ ) is combined with up to five additional particles (pions and/or kaons), as described in Ref. [6]. The  $B_{\text{tag}}$  candidates are required to have an energy-substituted mass  $m_{ES} \equiv \sqrt{s/4 - |\vec{p}_{\text{tag}}|^2} > 5.27 \text{ GeV}/c^2$ , and a difference between the beam energy and the reconstructed energy of the  $B_{\text{tag}}$  candidate  $|\Delta E| \equiv |E_{\text{tag}} - \sqrt{s}/2| \leq 0.09 \text{ GeV}$ , where  $\sqrt{s}$  is the total  $e^+e^-$  energy and  $\vec{p}_{\text{tag}}$  and  $E_{\text{tag}}$  are the measured  $B_{\text{tag}}$  momentum and energy in the  $e^+e^-$  CM frame.

For each  $B_{\text{tag}}$  candidate, we use the remaining particles in the event to search for signal  $\bar{B}$  meson candidates involving a  $D$  or  $D^*$  meson, a charged lepton, and up to two charged pions. The  $D^0$  and  $D^+$  candidates are reconstructed in final states involving up to four charged pions or kaons, up to one  $K_S^0 \rightarrow \pi^+ \pi^-$  decay, and up to one  $\pi^0 \rightarrow \gamma \gamma$  decay. We require  $1.845 < m(D^+) < 1.895 \text{ GeV}/c^2$  and  $1.840 < m(D^0) < 1.890 \text{ GeV}/c^2$ . The  $D^*$  mesons are reconstructed in  $D^{*0} \rightarrow D^0 \pi^0$ ,  $D^{*0} \rightarrow D^0 \gamma$ ,  $D^{*+} \rightarrow D^0 \pi^+$ , and  $D^{*+} \rightarrow D^+ \pi^0$  decays. Electrons and muons are identified using multivariate techniques based on information from the tracking detectors, calorimeter, and muon system, and are required to have a momentum larger than  $0.6 \text{ GeV}/c$  in the CM frame. We reject electrons consistent with photon conversions and Dalitz decays of  $\pi^0$  mesons. In cases where the flavor of the  $D^{(*)}$  meson is determined by its decay products, only combinations with the correct  $D^{(*)} \ell^-$  charge-flavor correlation are retained. For each  $B_{\text{tag}} D^{(*)} \ell^-$  candidate we allow up to two additional charged tracks in the event, resulting in a sample consisting of  $B_{\text{tag}} D^{(*)} (n\pi) \ell^-$  candidates, with “signal pion” multiplicity  $n = 0, 1$  or  $2$ . Our measurement is based on the  $n = 0$  and  $n = 2$  samples, while the  $n = 1$  sample is used to reject backgrounds in the  $n = 2$  sample.

Only candidates for which all charged tracks are assigned to one or the other  $B$  meson, and where the net charge of the event is zero, are considered

<sup>2</sup> The notation  $D^{(*)}$  denotes  $D^0$ ,  $D^+$ ,  $D^{*0}$  and  $D^{*+}$  mesons.

further. Charged  $B_{\text{tag}}$  candidates are required to have charge opposite that of the lepton candidate. We calculate  $E_{\text{extra}}$ , the energy sum of all calorimeter energy clusters with energy greater than 80 MeV that are not used in the reconstruction of the  $B$  candidates, and require  $E_{\text{extra}} \leq 0.4$  GeV. After these criteria are applied, the remaining events have on average about two  $\Upsilon(4S) \rightarrow B_{\text{tag}}\bar{B}$  candidates per signal channel. The candidate in each  $D^{(*)}(n\pi)\ell^-$  channel with the smallest  $|\Delta E|$  is retained.

Each  $\Upsilon(4S) \rightarrow B_{\text{tag}}\bar{B}$  candidate is fit to the hypothesized decay topology, imposing vertex and mass constraints on intermediate states in order to improve the resolution. The four-momentum of the  $B_{\text{tag}}D^{(*)}(n\pi)\ell^-$  candidate is subtracted from that of the initial  $e^+e^-$  state to determine the four-momentum  $p_{\text{miss}} = (E_{\text{miss}}, \vec{p}_{\text{miss}})$ . For events in which a single neutrino is the only missing particle, the difference  $U \equiv E_{\text{miss}} - |\vec{p}_{\text{miss}}|c$  peaks at zero with a resolution of  $\approx 0.1$  GeV;  $U$  is used to discriminate against events with additional missing particles. In contrast to the commonly used missing-mass-squared, which is proportional to  $E_{\text{miss}} + |\vec{p}_{\text{miss}}| \approx 2E_{\text{miss}}$ ,  $U$  does not depend directly on the modeling of  $E_{\text{miss}}$  and thus on the decay dynamics. Hadronic  $B$  decays for which all final-state particles are reconstructed, and in which a hadron is misidentified as an electron or muon, have  $E_{\text{miss}} \approx |\vec{p}_{\text{miss}}| \approx 0$ : we require  $|\vec{p}_{\text{miss}}| > 0.2$  GeV/ $c$  to suppress these events. We impose  $m(D^0\pi^\pm) - m(D^0) > 0.16$  GeV/ $c^2$  for the  $D^0\pi^+\pi^-\ell^-\bar{\nu}$  channel to remove correctly reconstructed  $B^- \rightarrow D^{*+}\pi^-\ell^-\bar{\nu}$  events with a subsequent  $D^{*+} \rightarrow D^0\pi^+$  decay.

We use a separate Fisher discriminant [16] in each signal channel to further reduce the background from continuum and  $B\bar{B}$  events. The variables used are  $E_{\text{extra}}$ ,  $m_{ES}$ , the number of unused neutral clusters with energy greater than 80 MeV, the numbers of charged tracks and neutral clusters in the  $B_{\text{tag}}$  candidate, the second normalized Fox-Wolfram moment  $R_2$  [17], and the CM-frame cosine of the angle between the thrust axes of the  $B_{\text{tag}}$  candidate and of the remaining particles in the event. The discriminants are constructed using simulated events, with the distribution of each variable reweighted to match the distribution in data. The selection requirement on the output variables is optimized assuming a branching fraction  $\mathcal{B}(\bar{B} \rightarrow D^{(*)}\pi^+\pi^-\ell^-\bar{\nu}) = 0.12\%$  in each channel.

At this stage of the analysis an event may be reconstructed in more than one channel. To obtain statistically independent samples and to maximize

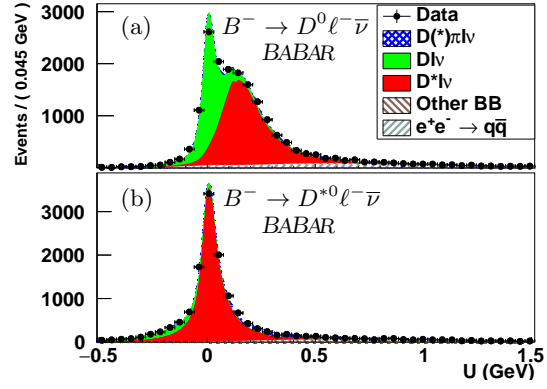


FIG. 1: Measured  $U$  distributions and results of the fit for the (a)  $B^- \rightarrow D^0\ell^-\bar{\nu}$  and (b)  $B^- \rightarrow D^{*0}\ell^-\bar{\nu}$  samples.

the sensitivity to  $D^{(*)}\pi^+\pi^-\ell^-\bar{\nu}$  decays, we select a unique candidate as follows. Any event found in a  $D^{(*)}\ell^-\bar{\nu}$  sample is removed from all samples with one or two signal pions. If an event enters two or more samples with the same number of signal pions, candidates are removed from the sample with lower signal-to-background level. In addition, we remove from the  $D^{(*)}\pi^+\pi^-\ell^-\bar{\nu}$  samples any event found in a  $D^{(*)}\pi\ell^-\bar{\nu}$  sample with  $|U| < 0.1$  GeV.

The analysis procedure was developed using simulated event samples; the data for the two-pion signal modes were not examined until the selection and fit procedures were finalized. Event yields are obtained from an unbinned maximum likelihood fit to the  $U$  distribution in the range  $-1.5 < U < 3.0$  GeV for each signal channel. One-dimensional probability density functions (PDF) for the signal and background components of each sample are obtained from MC using parametric kernel estimators with adaptive widths [18]. Figure 1 shows the results for the  $D^{(*)0}\ell^-\bar{\nu}$  channels; the results for the  $D^{(*)+}\ell^-\bar{\nu}$  channels are similar. Corresponding yields are presented in Table I.

The PDFs used in the fit to the  $D^{(*)}\ell^-\bar{\nu}$  channels include the following components, whose magnitudes are parameters of the fit:  $\bar{B} \rightarrow D\ell^-\bar{\nu}$ ,  $\bar{B} \rightarrow D^*\ell^-\bar{\nu}$ ,  $\bar{B} \rightarrow D^{(*)}\pi\ell^-\bar{\nu}$ , other  $B\bar{B}$  events, and continuum events. Potential contributions from  $D^{(*)}\pi\pi\ell^-\bar{\nu}$  decays have a similar shape to  $D^{(*)}\pi\ell^-\bar{\nu}$  decays in these channels and are included in the  $\bar{B} \rightarrow D^{(*)}\pi\ell^-\bar{\nu}$  component. The PDFs used in the fit to the  $D^{(*)}\pi^+\pi^-\ell^-\bar{\nu}$  channels include the following components:  $\bar{B} \rightarrow D^{(*)}\ell^-\bar{\nu}$ ,  $\bar{B} \rightarrow D^{(*)}\pi\ell^-\bar{\nu}$ ,  $\bar{B} \rightarrow$

$D\pi^+\pi^-\ell^-\bar{\nu}$ ,  $\bar{B} \rightarrow D^*\pi^+\pi^-\ell^-\bar{\nu}$ , other  $B\bar{B}$  events, and continuum events. Contributions to the  $\bar{B} \rightarrow D^{(*)}\pi^+\pi^-\ell^-\bar{\nu}$  channels from  $\bar{B} \rightarrow D^{(*)}\pi^\pm\pi^0\ell^-\bar{\nu}$  and  $\bar{B} \rightarrow D^{(*)}\pi^0\pi^0\ell^-\bar{\nu}$  decays (cross-feed) are treated as signal.

A fraction of signal decays are reconstructed with a  $B$  meson charge differing by  $\pm 1$  from the true  $B$  meson charge and contribute to the wrong signal channel. We determine this fraction for each signal channel in simulation and fix the corresponding yield ratio in the fit. Hadronic  $B$  meson decays in which a hadron is misidentified as a lepton can peak near  $U = 0$ . We estimate these small contributions using simulation and hold them fixed in the fit to the  $D^{(*)}\ell^-\bar{\nu}$  channels. Simulation indicates that these peaking backgrounds are negligible for the  $D^{(*)}\pi^+\pi^-\ell^-\bar{\nu}$  channels.

Fits to ensembles of parameterized MC pseudo-experiments are used to validate the fit. All fitted parameters exhibit unbiased means and variances.

The results for the  $D^{(*)}\pi^+\pi^-\ell^-\bar{\nu}$  channels are shown in Fig. 2 with the corresponding signal yields in Table I. The fitted yields for all background components are consistent with the values expected from MC. The only known source of  $\bar{B} \rightarrow D\pi^+\pi^-\ell^-\bar{\nu}$  decays is  $\bar{B} \rightarrow D_1(2420)\ell^-\bar{\nu}$  with  $D_1(2420) \rightarrow D\pi^+\pi^-$ . If we remove these  $D_1(2420)$  decays by vetoing events with  $0.5 < m(D\pi^+\pi^-) - m(D) < 0.6 \text{ GeV}/c^2$ , the signal yields are reduced to  $84.3 \pm 27.7$  events in  $D^0\pi^+\pi^-$ , and  $37.3 \pm 15.9$  in  $D^+\pi^+\pi^-$ , which indicates that  $D_1(2420) \rightarrow D\pi^+\pi^-$  is not the only source for the observed signals.

TABLE I: Event yields and estimated efficiencies ( $\epsilon$ ) for the signal channels. The quoted uncertainties are statistical only. The fourth column gives the statistical significance,  $\mathcal{S} = \sqrt{2\Delta\mathcal{L}}$ , where  $\Delta\mathcal{L}$  is the difference between the log-likelihood value of the default fit and a fit with the signal yield fixed to zero. The last column gives the total significance,  $\mathcal{S}_{tot}$ , where systematic uncertainties are included.

Channel	Yield	$\epsilon \times 10^4$	$\mathcal{S}$	$\mathcal{S}_{tot}$
$D^0\ell^-\bar{\nu}_\ell$	$5567 \pm 102$	$2.73 \pm 0.01$	$> 40$	$> 40$
$D^+\ell^-\bar{\nu}_\ell$	$3236 \pm 74$	$1.69 \pm 0.01$	$> 40$	$> 40$
$D^{*0}\ell^-\bar{\nu}_\ell$	$9987 \pm 126$	$2.03 \pm 0.01$	$> 40$	$> 40$
$D^{*+}\ell^-\bar{\nu}_\ell$	$5404 \pm 83$	$1.14 \pm 0.01$	$> 40$	$> 40$
$D^0\pi\pi\ell^-\bar{\nu}$	$171 \pm 30$	$1.18 \pm 0.03$	5.4	5.0
$D^+\pi\pi\ell^-\bar{\nu}$	$56 \pm 17$	$0.51 \pm 0.02$	3.5	3.0
$D^{*0}\pi\pi\ell^-\bar{\nu}$	$74 \pm 36$	$1.11 \pm 0.02$	1.8	1.6
$D^{*+}\pi\pi\ell^-\bar{\nu}$	$65 \pm 18$	$0.49 \pm 0.02$	3.3	3.0

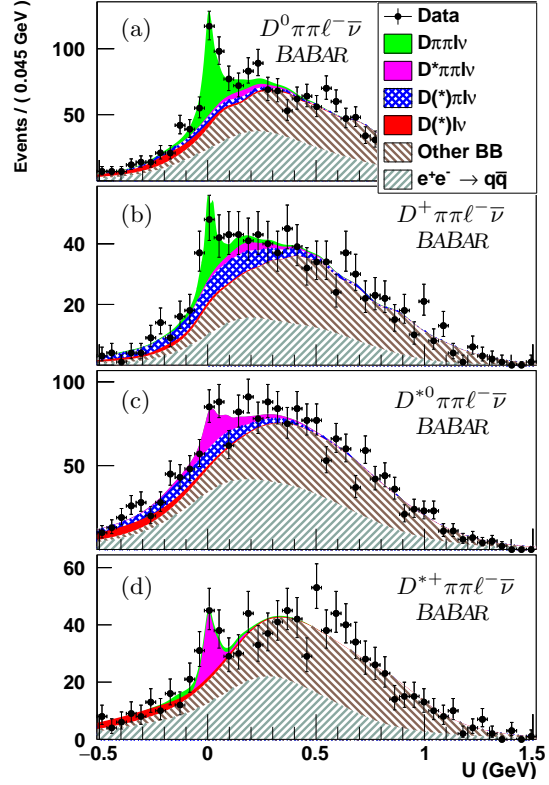


FIG. 2: Measured  $U$  distributions and results of the fit for the (a)  $D^0\pi\pi\ell^-\bar{\nu}$ , (b)  $D^+\pi\pi\ell^-\bar{\nu}$ , (c)  $D^{*0}\pi\pi\ell^-\bar{\nu}$ , and (d)  $D^{*+}\pi\pi\ell^-\bar{\nu}$  samples.

Systematic uncertainties arising from limited knowledge of branching fractions, form factors, and detector response are evaluated. These impact the determination of the PDF shapes, fixed backgrounds, cross-feed contributions, and signal efficiencies. The leading uncertainties arise from ignorance of potential resonance structure in the  $D^{(*)}\pi^+\pi^-$  final state, the limited size of MC samples used to derive PDFs, and the modeling of distributions of variables used in the Fisher discriminants. The dependence on the  $D^{(*)}\pi\pi$  production process is investigated by using, in turn, each of the individual mechanisms listed previously to model the signal. We assign the maximum deviation between the branching fraction ratios  $R_{\pi^+\pi^-}^{(*)}$  obtained from the nominal and alternative decay models as an uncertainty, giving 7.8% for  $D^0\pi^+\pi^-\ell^-\bar{\nu}$ , 10.5% for  $D^+\pi^+\pi^-\ell^-\bar{\nu}$ , 19.2% for  $D^{*0}\pi^+\pi^-\ell^-\bar{\nu}$ , and 13.4% for  $D^{*+}\pi^+\pi^-\ell^-\bar{\nu}$ . The impact of the statistical uncertainties of the PDFs are estimated from fits to 1300 simulated data sets, obtained from

the primary MC samples using the bootstrapping method [19], resulting in uncertainties ranging from 6.5% ( $D^0\pi^+\pi^-\ell^-\bar{\nu}$ ) to 21.1% ( $D^{*0}\pi^+\pi^-\ell^-\bar{\nu}$ ). We estimate the uncertainty associated with modeling the Fisher discriminants by using the uncorrected shape of each simulated input distribution, one at a time, before imposing the selection requirement. The systematic uncertainty, given by the sum in quadrature of the differences with respect to the nominal analysis, varies from 3.7% ( $D^0\pi^+\pi^-\ell^-\bar{\nu}$ ) to 5.2% ( $D^+\pi^+\pi^-\ell^-\bar{\nu}$ ).

The ratios of branching fractions are calculated from the fitted yields as

$$R_{\pi^+\pi^-}^{(*)} = \frac{N_{\pi^+\pi^-}^{(*)}}{N_{\text{norm}}^{(*)}} \frac{\epsilon_{\text{norm}}^{(*)}}{\epsilon_{\pi^+\pi^-}^{(*)}}, \quad (1)$$

where  $\epsilon$  refers to the corresponding efficiency, which is calculated from MC for the same type of  $B$  meson ( $B^-$  or  $\bar{B}^0$ ) used in the two-pion signal ( $N_{\pi^+\pi^-}^{(*)}$ ) and zero-pion normalization ( $N_{\text{norm}}^{(*)}$ ) yields. The results are given in Table II. The dependence of the efficiencies on the details of the hadronic  $B$  reconstruction largely cancels in the ratio, as do some other associated systematic uncertainties and possible biases. Since semileptonic  $B$  decays proceed via a spectator diagram, the semileptonic decay widths of neutral and charged  $B$  mesons are expected to be equal. We therefore determine combined values for the  $B^-$  and  $\bar{B}^0$  channels: these are given in Table II. Also shown are the corresponding  $B^-$  branching fractions obtained by using Ref. [4] for the branching fractions of the normalization modes.

TABLE II: Branching fraction ratios  $R_{\pi^+\pi^-}^{(*)}$  for the  $D^{(*)}\pi^+\pi^-\ell^-\bar{\nu}$  channels and corresponding isospin-averaged values. The first uncertainty is statistical and the second is systematic. The rightmost column gives the corresponding branching fractions, where the third uncertainty comes from the branching fraction of the normalization mode. The isospin-averaged results are quoted as  $B^-$  branching fractions.

Channel	$R_{\pi^+\pi^-}^{(*)} \times 10^3$	$\mathcal{B} \times 10^5$
$D^0\pi^+\pi^-\ell^-\bar{\nu}$	$71 \pm 13 \pm 8$	$161 \pm 30 \pm 18 \pm 8$
$D^+\pi^+\pi^-\ell^-\bar{\nu}$	$58 \pm 18 \pm 12$	$127 \pm 39 \pm 26 \pm 7$
$D^{*0}\pi^+\pi^-\ell^-\bar{\nu}$	$14 \pm 7 \pm 4$	$80 \pm 40 \pm 23 \pm 3$
$D^{*+}\pi^+\pi^-\ell^-\bar{\nu}$	$28 \pm 8 \pm 6$	$138 \pm 39 \pm 30 \pm 3$
$D\pi^+\pi^-\ell^-\bar{\nu}$	$67 \pm 10 \pm 8$	$152 \pm 23 \pm 18 \pm 7$
$D^*\pi^+\pi^-\ell^-\bar{\nu}$	$19 \pm 5 \pm 4$	$108 \pm 28 \pm 23 \pm 4$

In conclusion, the decays  $\bar{B} \rightarrow D^{(*)}(n\pi)\ell^-\bar{\nu}$  with  $n = 0$  or 2 are studied in events with a fully reconstructed second  $B$  meson. We obtain the first observation of  $\bar{B} \rightarrow D^0\pi^+\pi^-\ell^-\bar{\nu}$  decays and first evidence for  $\bar{B} \rightarrow D^{(*)+}\pi^+\pi^-\ell^-\bar{\nu}$  decays. The branching ratios of  $\bar{B} \rightarrow D^{(*)}\pi^+\pi^-\ell^-\bar{\nu}$  decays relative to the corresponding  $\bar{B} \rightarrow D^{(*)}\ell^-\bar{\nu}$  decays are measured. To estimate the total  $\bar{B} \rightarrow D^{(*)}\pi\pi\ell^-\bar{\nu}$  branching fraction we use isospin symmetry and consider in turn each of the  $\bar{B} \rightarrow X_c\ell^-\bar{\nu}$  decay models discussed above. We find  $\mathcal{B}(\bar{B} \rightarrow D^{(*)}\pi^+\pi^-\ell^-\bar{\nu})/\mathcal{B}(\bar{B} \rightarrow D^{(*)}\pi\pi\ell^-\bar{\nu}) = 0.50 \pm 0.17$ , where the uncertainty is one half the observed spread from the investigated models, which implies  $\mathcal{B}(\bar{B} \rightarrow D\pi\pi\ell^-\bar{\nu}) + \mathcal{B}(\bar{B} \rightarrow D^*\pi\pi\ell^-\bar{\nu}) = (0.52_{-0.07-0.13}^{+0.14+0.27})\%$ , where the first uncertainty is the total experimental uncertainty and the second is due to the unknown fraction of  $\bar{B} \rightarrow D^{(*)}\pi^+\pi^-\ell^-\bar{\nu}$  in  $\bar{B} \rightarrow D^{(*)}\pi\pi\ell^-\bar{\nu}$  decays. This corresponds to between one-quarter and one-half of the difference between the sum of the previously measured exclusive  $B$  meson semileptonic decays to charm final states and the corresponding inclusive semileptonic branching fraction.

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